* برهامي

* الاسبوع الاول ، محاضرة 1

* P.D.E
  - → 1^{st} order
  - → 2^{nd} order

* harmonic analysis:
  1) fourier series
  2) fourier transform
  3) laplace transform

* Vector differential operator ($\bar{\nabla}$).

$$\bar{\nabla}\ (\text{nabla}) = \frac{\partial}{\partial x}\hat{i} + \frac{\partial}{\partial y}\hat{j} + \frac{\partial}{\partial z}\hat{k}$$
$$= \partial_x \hat{i} + \partial_y \hat{j} + \partial_z \hat{k}$$

• **Scaler function:** (scaler field $\xrightarrow{ex}$ current density $f = f(x,y,z)$)

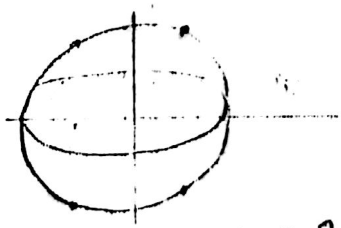
* $\phi = x^2 + y^2 + xy + \sin z \rightarrow \phi(x,y,z) = 0$ مش شرط صفر قيمة معينة وثابتة

* دالة تعتمد على موضع النقطة، وقيمتها تكون ثابتة.

* Sphere: $Q(x,y,z) = x^2 + y^2 + z^2 - R^2 = 0$

scaler function

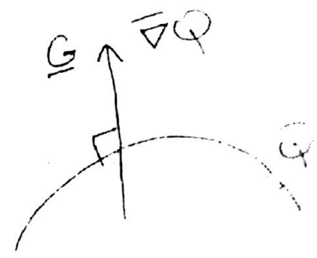
لو قيمة ϕ تساوي قيمة ثابتة



• **Vector function:** (vector field $\xrightarrow{ex}$ electric field $\bar{E} = E_x\hat{i} + E_y\hat{j} + E_z\hat{k}$
$\xrightarrow{ex}$ magnetic field $\bar{H} = H_x\hat{i} + H_y\hat{j} + H_z\hat{k}$)

$$\bar{F} = F_1(x,y,z)\hat{i} + F_2(x,y,z)\hat{j} + F_3(x,y,z)\hat{k}$$

$\xrightarrow{ex}$ $\bar{F} = \underbrace{(x^2+y^2+z^2)}_{F_1}\hat{i} + \underbrace{e^{xyz}}_{F_2}\hat{j} + \underbrace{x^2 y}_{F_3}\hat{k}$



• **gradient of scaler function (field):**

$$\text{Grad}\ Q = \bar{\nabla} Q = \frac{\partial Q}{\partial x}\hat{i} + \frac{\partial Q}{\partial y}\hat{j} + \frac{\partial Q}{\partial z}\hat{k}$$

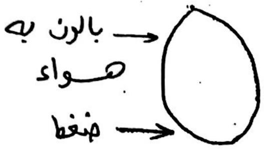
* بالتجاه العمودي على سطح (scaler function)

- **Divergence:-** التباين

$$\text{Div } F = \overline{\nabla} \odot \overline{F} \neq \overline{F} \odot \overline{\nabla}$$ (ضرب قياسي)

$$= \frac{\partial F_1}{\partial x} + \frac{\partial F_2}{\partial y} + \frac{\partial F_3}{\partial z}$$

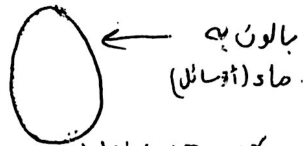
- **example:-**



يسمح بالانضغاط

$\text{Div} \neq 0$

Non-Solenoidal field



لا يسمح بالانضغاط

$\text{Div} = 0$

Solenoidal field

- **Rotation [Curl]:-** الدوران

$$\text{Curl}(\overline{F}) = \overline{\nabla} \times \overline{F}$$ (ضرب اتجاهي)

$$= \begin{vmatrix} i & j & k \\ \frac{\partial}{\partial x} & \frac{\partial}{\partial y} & \frac{\partial}{\partial z} \\ F_1 & F_2 & F_3 \end{vmatrix}$$

→ when $\text{Curl} = 0 \therefore \overline{F}$ is irrotational field (غير دوراني)

→ when $\text{Curl} \neq 0 \therefore \overline{F}$ is rotational field (دوراني)

- **LapLacion operator (Δ):-**

$$\Delta = \overline{\nabla} \odot \overline{\nabla}$$

$$= (\partial_x \hat{i} + \partial_y \hat{j} + \partial_z \hat{k}) \odot (\partial_x \hat{i} + \partial_y \hat{j} + \partial_z \hat{k})$$

$$= \partial_{xx} + \partial_{yy} + \partial_{zz}$$

$$= \frac{\partial^2}{\partial x^2} + \frac{\partial^2}{\partial y^2} + \frac{\partial^2}{\partial z^2}$$

* Harmonic function:- دالة توافقية

if $\Delta Q = 0$

$\therefore$ Q is a harmonic function

ex $\rightarrow$ $Q = x^2 - y^2$

$\frac{\partial^2 Q}{\partial x^2} = 2$; $\frac{\partial^2 Q}{\partial y^2} = -2 \longrightarrow \Delta = \frac{\partial^2 Q}{\partial x^2} + \frac{\partial^2 Q}{\partial y^2} = 2 - 2 = 0$

$\therefore$ Q is a harmonic function.

* what is P.D.E :-

- an equation which contains the unknown function and its partial derivatives.

ex $\rightarrow$ $u_x + u_y = 0$, $u_x(x,y,z) = 0$, $(u_x)^2 + u_{xx} + u_{yy} = 0$

.. order of P.D.E:- highest ordered-derivative in the equation.

.. Degree of P.D.E:- The power of the highest ordered-derivative in the equation.

ex $\rightarrow$ 1) $u_{xxx} + (\Delta u)^2 = 0$ $\rightarrow$ order (3), degree (1)

2) $\frac{\partial^{3/2} u}{\partial x^{3/2}} + (u_y)^2 = 0$ $\rightarrow$ order $(\frac{3}{2})$, degree (1)

** The General P.D.E of 1st order in two dimensions:-

$F(x, y, u, u_x, u_y) = 0$

ex $\rightarrow$ $(u_x)^2 + (u_y)^2 + xu = 0$ order (1) , degree (2)

** The General P.D.E of 2nd order in two dimensions:-

$F(x, y, u, u_x, u_y, u_{xx}, u_{xy}, u_{xy}) = 0$

ex $\rightarrow$ $(u_{xy})^2 + u_{xx} + u_y = 0$ order (2), degree (2)

Types of 2^{nd} order P.D.E :-

① Linear 2^{nd} order PDE :-

$$A(x,y)\,u_{xx}+B(x,y)\,u_{xy}+C(x,y)\,u_{yy}+D(x,y)\,u_x+E(x,y)\,u_y+F(x,y)\,u=G(x,y)$$

[A, B, C, D, E, F, G] are functions in x, y or constants.

if $G=0 \longrightarrow$ homogenous
$G\neq 0 \longrightarrow$ non-homogenous

② Semi-Linear 2^{nd} order P.D.E :-

$$A(x,y)\,u_{xx}+B(x,y)\,u_{xy}+C(x,y)\,u_{yy}+D(x,y)\,u_x+E(x,y)\,u_y=G(x,y,u)$$

(F, G) has non-Linear relation to u $\xrightarrow{ex} G=u^2xy,\ xe^u,\ \sin u$

$\xrightarrow{ex} u_{xx}+u_{yy}+u=u^2 \rightarrow u_{xx}+u_{yy}=(u^2-u) \leftarrow G$

③ ALmost Linear 2^{nd} order P.D.E :-

$$A(x,y)\,u_{xx}+B(x,y)\,u_{xy}+C(x,y)\,u_{yy}+G(x,y,u,u_x,u_y)=0$$

(A, B, C) Linear , (D, E, F, G) non-linear

$\xrightarrow{ex} u_{yy}+(u_x)^2+u=0 \rightarrow u_{yy}+[u+(u_x)^2]=0$ ← G non-Linear relation

④ Quasi-Linear 2^{nd} order P.D.E :-

$$A(x,y,u,u_x,u_y)\,u_{xx}+B(x,y,u,u_y,u_x)\,u_{xy}+C(x,y,u,u_x,u_y)\,u_{yy}+G(x,y,u,u_x,u_y)=0$$

$\xrightarrow{ex} u_x u_{xx}+(u_y)^2=0$

⑤ Non-Linear 2^{nd} order P.D.E :-

It is not any of the previous cases.

• تنتج اذا كان A, B, C يحتوى على أحد المتغيرات التفاضلية u_{xx}, u_{xy}, u_{yy}

$\xrightarrow{ex} (u_{xy})^2+2u_x+3u^2=0$